Calc AB Ch 6 Test Practice

Name: Block: Seat:

Chapter 6 Check List:

1 ..... Sketching and identifying a Slope Field (p. 408)

2 ..... Sketching a solution of a differential equation using a slope field (p 409)

3 ..... Solving a differential equations (p 415)

4 ..... Growth and decay models solving $\frac{dy}{dt} = ky$ (p 416)

5 ..... Finding general and particular solutions to separable differential equations. (p 434-4)

Delta Math Check List:

1 ..... Practice Slope Fields (6.1)

2 ..... Practice Diff-EQs (6.2-6.3)

Khan Academy Check List:

1 ..... Differential Equations Unit: 7 lessons, 2 Quizzes

2 ..... Explanation of 2011 AB 5 Videos

Always review your Notes and Examples (see topics if you lost your notes), Quizzes, and old homework problems. There is a separate pdf with Multiple choice practice as well.

1. (Calc OK))A sample of Dl-13 (an isotope of Delerium) loses 99% of its radioactive matter in 199 hours. What is the half-life of Dl-13?
   (a) 4 hours
   (b) 6 hours
   (c) 30 hours
   (d) 100.5 hours
   (e) 143 hours
   (f) None of these

2. In which of the following models is $\frac{dy}{dt}$ directly proportional to y?

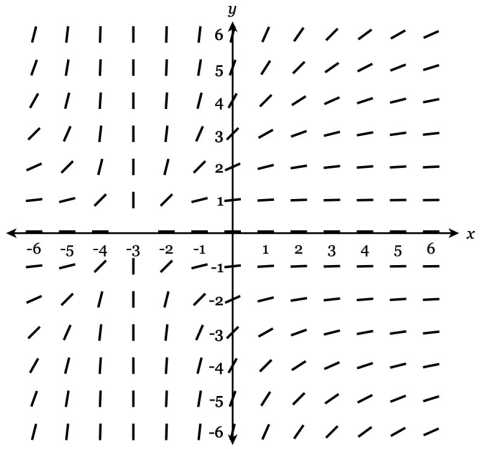
   I $y = e^{kt} + C$
   II $y = Ce^{kt}$
   III $y = 28^{kt}$
   IV $y = 3\left(\frac{1}{2}\right)^{3t+1}$

   (a) I only
   (b) IIonly
   (c) I and II only
   (d) II and III only
   (e) II, III, and IV only
   (f) All four

3. (Calculator Active) The rate at which acreage is being consumed by a plot of kudzu is proportional to the number of acres already consumed at time t. If there are 2 acres consumed when $t = 1$ and 3 acres consumed when $t = 5$, how many acres will be consumed when $t = 8$?
   (a) 3.750
   (b) 4.000
   (c) 4.066
   (d) 4.132
   (e) 4.600
   (f) None of these

4. Which of the following are separable differential equations? (There may be more than one)
   (a) $\frac{dy}{dx} = \ln(x + y)$
   (b) $\frac{dy}{dx} = e^{x+y}$
   (c) $\frac{dy}{dx} = x^2 + 3xy$
   (d) $\frac{dy}{dx} = xy + 4x$
   (e) None of these

5. Select the differential equation that matches the given slope field.



(a) $\frac{dy}{dx} = -\frac{y}{x+3}$

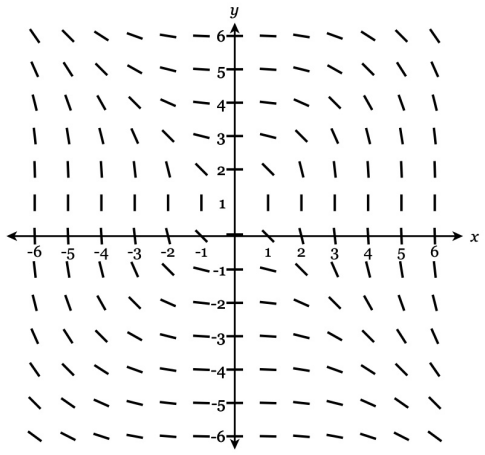
(b) $\frac{dy}{dx} = (x+3)y^2$

(c) $\frac{dy}{dx} = \frac{x+3}{y}$

(d) $\frac{dy}{dx} = \frac{y^2}{(x+3)^2}$

(e) None of these

6. Select the differential equation that matches the given slope field.



(a) $\frac{dy}{dx} = \frac{(y-1)^2}{x}$

(b) $\frac{dy}{dx} = -\frac{x}{y-1}$

(c) $\frac{dy}{dx} = -\frac{x^2}{(y-1)^2}$

(d) $\frac{dy}{dx} = -\frac{y-1}{x}$

(e) None of these

7. Which of the following is a solution of the differential equation $xy' - 4y = x^5e^x$?
   (a) $y = 4x^5e^{2x}$
   (b) $y = 6e^{2x} - 7\sin 2x$
   (c) $y = x^4e^x$
   (d) $y = 5e^{-2x}$
   (e) $y = \ln x$
   (f) None of these

8. (a) (3 points) Consider the differential equation $\dfrac{dy}{dx} = xy^3$ with a particular solution $y = f(x)$ having an initial condition $y(-2) = -1$. Use the equation of the line tangent to the graph of f at the point $(-2, -1)$ in order to approximate the value of $f(-1.9)$.

   (b) (3 points) Consider the differential equation $\dfrac{dy}{dx} = (x^2+3)(y-2)$ with a particular solution $y = f(x)$ having an initial condition $y(0) = -3$. Use the equation of the line tangent to the graph of f at the point $(0, -3)$ in order to approximate the value of $f(0.1)$.

9. Find the general solution to the following differential equations, then find the particular solution using the initial condition.

   (a) $\dfrac{dy}{dx} = \dfrac{x}{y}, \quad y(1) = -2$

   (b) $\dfrac{dy}{dx} = -\dfrac{x}{y}, \quad y(4) = 3$

   (c) $\dfrac{dy}{dx} = \dfrac{y}{x}, \quad y(2) = 2$

   (d) $\dfrac{dy}{dx} = 2xy, \quad y(0) = -3$

(e) $\dfrac{dy}{dx} = (y+5)(x+2), \quad y(0) = -1$

(f) $\dfrac{dy}{dx} = \cos^2(y), \quad y(0) = 0$

(g) $\dfrac{dy}{dx} = (\cos x)e^{y+\sin x}, \quad y(0) = 0$

(h) $\dfrac{dy}{dx} = e^{x-y}, \quad y(0) = 2$

(i) $\dfrac{dy}{dx} = -2xy^2, \quad y(1) = \dfrac{1}{4}$

(j) $\dfrac{dy}{dx} = \dfrac{4\sqrt{y}\ln x}{x}, \quad y(e) = 1$

10. Find the solution of the differential equation $\frac{dy}{dt} = ky$ that satisfies the given conditions.

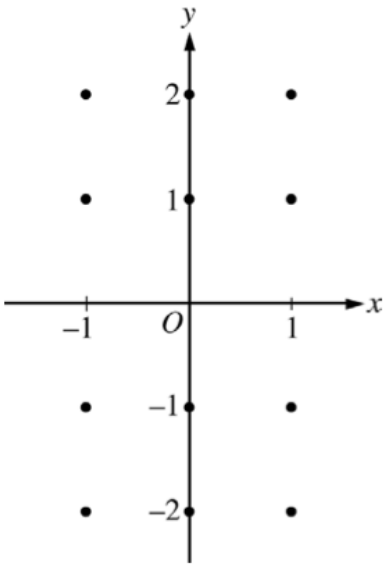
    (a) $y(0) = 50$ and $y(5) = 100$

    (b) The graph of y passes through $(1, 55)$ and $(10, 30)$

11. Write and find a general solution of the differential equation that describes this statement: The rate of change of G with respect to to t is proportional to $50 - t$.

12. (2010B AB 5 - No Calc) Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) (3 points) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

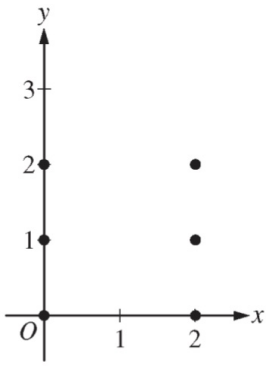


(b) (1 point) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $dy = -1$.

(c) (5 points) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$

13. (2016 AB 4 - No Calc) Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) (2 points) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

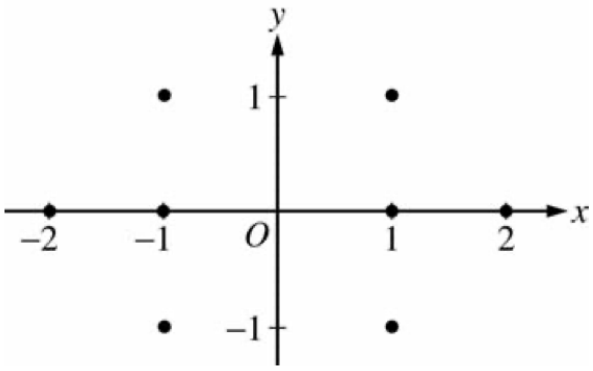


(b) (2 points) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Us your equation to approximate $f(2.1)$.

(c) (5 points) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

14. (2006 AB 5 - No Calc) Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$ where $x \neq 0$

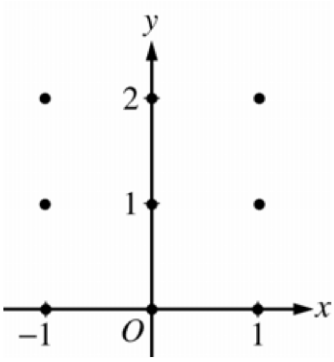
(a) (2 points) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



(b) (7 points) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain..

15. (2007B AB 5 - No Calc) Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

(a) (2 points) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



(b) (3 points) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) (2 points) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) (2 points) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.